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On ps-ro semiopen fuzzy set and ps-ro fuzzy semicontinuous, semiopen functions

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Abstract

The aim of this paper is to introduce and characterize ps-ro semiopen (semiclosed) fuzzy sets, which are totally independent of the existing notion of fuzzy open (closed) sets and semiopen (semiclosed) fuzzy sets. Also, in term of these fuzzy sets and operators ps-scl and ps-sint, a class of functions named as ps-ro fuzzy semicontinuous and ps-ro fuzzy semiopen (closed) functions are defined and their various properties are studied. ps-ro fuzzy semicontinuity is indeed totally different from both the existing concepts of fuzzy continuity and fuzzy semicontinuity. Similarly, ps-ro fuzzy open (closed) and well known concept of fuzzy semiopen (closed) functions do not imply each other. These concepts are used as new tools to study different characterizations of the given fuzzy topological space, giving a new dimension in the study of fuzzy topological spaces.

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1 Introduction and Preliminaries

Ever since the introduction of fuzzy logic and fuzzy sets by L.A.Zadeh [13] and fuzzy topology and fuzzy continuity by C. L. Chang [2], there have been several generalizations and also various departures of different notions of general topological concepts in their fuzzy settings. K. K Azad [1] introduced and studied the concept of fuzzy semiopen(closed) sets, fuzzy semiopen(closed) functions and fuzzy semicontinuity. Some relevant development on these topics are found in [9], [6], etc.

In a fuzzy topological space (fts, for short) (X, τ) , the family $i_{\alpha}(\tau) = \{\mu^{\alpha} : \mu \in \tau\}$ for all $\alpha \in I_1 = [0,1)$ where $\mu^{\alpha} = \{x : \mu(x) \geq \alpha\}$ forms a topology called strong α -level topology on X. The study of strong α -level topology on X reveals various facts about the fts (X, τ) [8], [7]. In [3], it was observed that a strong α -level set μ^{α} need not be regular open on $(X, i_{\alpha}(\tau))$ for all $\alpha \in I_1$ if μ is fuzzy regular open on (X, τ) and also the regular openness of μ^{α} may fail to imply the fuzzy regularity of μ . This observation gave birth to a new family of fuzzy sets termed as pseudo regular open(closed) fuzzy sets on X. The family of all pseudo regular open fuzzy sets form a fuzzy topology on X which is coarser than τ and is termed as ps-ro fuzzy topology on X. Members of ps-ro fuzzy topology are called ps-ro open fuzzy sets and their complements are ps-ro closed fuzzy sets on (X, τ) . In [4], interms of above ps-ro fuzzy sets, a fuzzy continuous type function called ps-ro fuzzy continuous function was introduced and different properties were studied.

In this paper, with the help of above mention ps-ro open fuzzy sets, a new class of fuzzy sets called ps-ro semiopen(semiclosed) fuzzy set is introduced and studied. It is seen that ps-ro semiopen (semiclosed) fuzzy sets are totally independent not only from the existing concepts of semiopen(closed) fuzzy sets but also from fuzzy open(closed) sets. Operator ps-scl is defined with

the help of ps-semicluster point. Another similar operator ps-int is introduced and different characterizations and interrelations are studied. Further, in terms of these fuzzy sets and operators, concept of ps-ro fuzzy semicontinuous, ps-ro fuzzy semiopen(semiclosed) functions are introduced and their various properties are studied. It is observed that every ps-ro fuzzy continuous function is ps-ro fuzzy semicontinuous but not conversely. Every ps-ro fuzzy semicontinuous functions are totally different from well known concepts of fuzzy continuous as well as fuzzy semicontinuous functions. Also, a characterization of such function is made in terms of graph of the function in product topology. It was also seen that ps-ro fuzzy semiopen(closed) and fuzzy semiopen(closed)functions do not imply each other. It is worth mentioning here that the above concepts of ps-ro fuzzy sets and different functions are used as new tools to study different characterizations of the given fuzzy topological space, giving a new dimension in the study of fts.

To make this paper self content, we state a few known definitions and results here that we require subsequently.

Let X be a non-empty set and I be the closed interval [0,1]. A fuzzy set μ on X is a function on X into I. If f is a fuction from X into a set Y and A, B are fuzzy sets on X and Y respectively, then 1-A (called complement of A), f(A) and $f^{-1}(B)$ are fuzzy sets on X, Y and X respectively,

defined by
$$(1 - A)(x) = 1 - A(x) \forall x \in X$$
, $f(A)(y) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), when f^{-1}(y) \neq \emptyset \\ 0, otherwise \end{cases}$ and

 $f^{-1}(B)(x) = B(f(x))$ [13]. A collection $\tau \subseteq I^X$ is called a fuzzy topology on X if (i) $0, 1 \in \tau$ (ii) $\forall \mu_1, \mu_2, ..., \mu_n \in \tau \Rightarrow \wedge_{i=1}^n \mu_i \in \tau \text{ (iii) } \mu_\alpha \in \tau, \forall \alpha \in \Lambda \text{ (where } \Lambda \text{ is an index set)} \Rightarrow \vee \mu_\alpha \in \tau. \text{ Then,} (X, \tau) \text{ is called a } fts$ [2]. A fuzzy set is called a fuzzy point, denoted by x_α , where $0 < \alpha \le 1$, and

defined as
$$x_{\alpha}(z) = \begin{cases} \alpha, & \text{for } z = x \\ 0, & \text{otherwise.} \end{cases}$$
. A fuzzy point x_{α} is said to q-coincident with a fuzzy set A ,

denoted by $x_{\alpha}qA$ if $\alpha + A(x) > 1$. If A and B are not q-coincident, we write A βB . A fuzzy set A is said to be a q-neighbourhood (or, q-nbd.) of a fuzzy point x_{α} if there is a fuzzy open set B such that $x_{\alpha}qB \leq A$ [10]. A set S in a topological space (X,τ) is called regular open if S = int(clS) [11]. Let f be a function from a set X into a set Y. Then the following holds:

- (i) $f^{-1}(1-B) = 1 f^{-1}(B)$, for any fuzzy set B on Y.
- (ii) $A_1 \leq A_2 \Rightarrow f(A_1) \leq f(A_2)$, for any fuzzy sets A_1 and A_2 on X. (iii) $B_1 \leq B_2 \Rightarrow f^{-1}(B_1) \leq f^{-1}(B_2)$, for any fuzzy sets B_1 and B_2 on Y.
- (iv) $ff^{-1}(B) \leq B$, for any fuzzy set B on Y and the equality holds if f is onto. (v) $f^{-1}f(A) \geq A$, for any fuzzy set A on X, equality holds if f is one-to-one [2]. A fuzzy set μ on a fts (X, τ) is called fuzzy regular open if $\mu = int(cl\mu)$. A fuzzy set A on a fts (X, τ) is said to be semiopen fuzzy set if there exist a fuzzy open set U such that $U \leq A \leq cl(U)$. A is said to be semiclosed fuzzy set if there exist a fuzzy closed set V such that $int(V) \leq A \leq V$. For a function $f: X \to Y$, the graph $g: X \to X \times Y$ of f is defined by g(x) = (x, f(x)), for each $x \in X$, where X and Y are any sets. Let (X, τ_1) and (X, τ_2) be two fts. The fuzzy product space of X and Y is the cartesian product $X \times Y$ of sets together with the fuzzy topology $\tau_1 \times \tau_2$ generated by the family $\{(p_1^{-1}(A), p_2^{-1}(B)) : A \in \tau_1, B \in \tau_2\}, \text{ where } p_1 \text{ and } p_2 \text{ are the projections of } X \times Y \text{ on } X \text{ and } Y$ respectively. The family $B = \{A \times B : A \in \tau_1, B \in \tau_2\}$ forms a base for the product topology $\tau_1 \times \tau_2$ on $X \times Y$ [1]. A function f from a fts (X, τ) to fts (Y, σ) is said to be
- (i) fuzzy continuous [2], if $f^{-1}(\mu)$ is fuzzy open on X, for all fuzzy open set μ on Y.
- (ii) fuzzy open (fuzzy closed) [12], if for each fuzzy open (fuzzy closed) set μ on X, $f(\mu)$ is fuzzy open (fuzzy closed) on Y.

(iii)fuzzy semicontinuous [1], if $f^{-1}(\mu)$ is semiopen fuzzy set on X, for all fuzzy open set μ on Y. (iv) fuzzy semiopen (fuzzy semiclosed) [1], if for each fuzzy open (fuzzy closed) set μ on X, $f(\mu)$ is semiopen fuzzy set (semiclosed fuzzy) on Y.

For a fuzzy set μ in X, the set $\mu^{\alpha} = \{x \in X : \mu(x) > \alpha\}$ is called the strong α -level set of X. In a fts (X,τ) , the family $i_{\alpha}(\tau) = \{\mu^{\alpha} : \mu \in \tau\}$ for all $\alpha \in I_1 = [0,1)$ forms a topology on X called strong α -level topology on X [8], [7]. A fuzzy open(closed) set μ on a fts (X,τ) is said to be pseudo regular open(closed) fuzzy set if the strong α -level set μ^{α} is regular open(closed) in $(X,i_{\alpha}(\tau)), \forall \alpha \in I_1$. The family of all pseudo regular open fuzzy sets form a fuzzy topology on X called ps-ro fuzzy topology on X which is coarser than τ . Members of ps-ro fuzzy topology are called ps-ro open fuzzy sets and their complements as ps-ro closed fuzzy sets on (X,τ) [3]. A function f from fts (X,τ_1) to fts (Y,τ_2) is pseudo fuzzy ro continuous (in short, ps-ro fuzzy continuous) if $f^{-1}(U)$ is ps-ro open fuzzy set on X for each pseudo regular open fuzzy set U on Y [4]. Equivalently, f is ps-ro fuzzy continuous if $f^{-1}(A)$ is ps-ro open fuzzy set on X for each ps-ro open fuzzy set A on A for each A on a A function of all A is denoted by A on A for each A on a A fuzzy A on A function of A and is denoted by A is contained in a fuzzy set A on a A is A is called fuzzy A fuzzy A is contained by A is A in A

Theorem 1.1. [4], [5] For any fuzzy sets A and B on a fts (X, τ) , the following hold: (i) ps-cl(A) is the smallest ps-ro closed fuzzy set containing A and ps-int(A) is the largest ps-ro open fuzzy set contained in A.

- (ii) $ps\text{-}cl(A) \leq ps\text{-}cl(B)$ and $ps\text{-}int(A) \leq ps\text{-}int(B)$ if $A \leq B$.
- (iii) $ps-cl(A) = A \ iff \ A \ is \ ps-ro \ closed \ and \ ps-int(A) = A \ iff \ A \ is \ ps-ro \ open.$
- (iv) ps-cl(ps-cl(A)) = ps-cl(A) and ps-int(ps-int(A)) = ps-int(A).
- (v) $ps\text{-}cl(A \lor B) = ps\text{-}cl(A) \lor ps\text{-}cl(B)$ and $ps\text{-}int(A \land B) = ps\text{-}int(A) \land ps\text{-}int(B)$.
- (vi)1 ps-intA = ps-cl(1 A) and 1 ps-clA = ps-int(1 A).

2 ps-ro semiopen(closed) fuzzy set and ps operators

Definition 2.1. A fuzzy set A on a fts (X,τ) is said to be ps-ro semiopen fuzzy set if there exist a ps-ro open fuzzy set U such that $U \le A \le ps$ -cl(U). A is said to be ps-ro semiclosed fuzzy set if there exist a ps-ro closed fuzzy set V such that ps- $int(V) \le A \le V$.

Remark 2.1. Clearly, on a $fts(X,\tau)$ every ps-ro open(closed) fuzzy set is fuzzy open(closed) set. Also, every ps-ro open(closed) fuzzy set is ps-ro semiopen(semiclosed). But the converse is not always true is shown by the following Example:

Example 2.1. Let $X = \{a, b, c\}$ and A, B and C be three fuzzy sets on X defined by A(x) = 0.1 $\forall x \in X, B(a) = 0.2, B(b) = 0.2$ and B(c) = 0.3 and C(x) = 0.4 $\forall x \in X$. It is easy to verify that $\tau = \{0, 1, A, B\}$ is a fuzzy topology on X. B is not pseudo regular open fuzzy set as B^{α} is not regular open on the topological space $(X, i_{\alpha}(\tau))$ for $0.2 \le \alpha < 0.3$, although B is fuzzy open on (X, τ) . Here, the collection of all pseudo regular open fuzzy sets on X is $\{0, 1, A\}$. Hence, ps-ro fuzzy topology on X is $\{0, 1, A\}$ which is strictly coarser than τ . Here, B is fuzzy open but is not ps-ro open fuzzy set on the fts (X, τ) . 1 - B is fuzzy closed but is not ps-ro closed fuzzy set on the fts (X, τ) . Now, ps-cl(A) = 1 - A, where (1 - A)(x) = 0.9 $\forall x \in X$. So, $A \le C \le \text{ps-cl}(A)$. Hence, C is a ps-ro semiopen fuzzy set but it is neither fuzzy open nor ps-ro open fuzzy set on the fts (X, τ) . Similarly, 1 - C is a ps-ro semiclosed fuzzy set but it is neither fuzzy closed nor ps-ro closed fuzzy set on the fts (X, τ) .

Example 2.2. Let $Y = \{a, b, c\}$. Let A, B, C, D, E and F be fuzzy sets on Y defined by A(y) = 0.3 $\forall y \in Y, B(a) = 0.1, B(b) = 0.1$ and B(c) = 0.2, C(y) = 0.2 $\forall y \in Y, D(y) = 0.4$ $\forall y \in Y, E(a) = 0.5, E(b) = 0.6$ and E(c) = 0.6 and F(y) = 0.5 $\forall y \in Y$. Then, $\tau_1 = \{0, 1, A, B, D, E\}$ is a fuzzy topology on Y. Also, B and E are not pseudo regular open fuzzy set for $0.1 \le \alpha < 0.2$ and $0.5 \le \alpha < 0.6$ respectively. Clearly, ps-ro topology on Y is $\{0, 1, A, D\}$. Here, $B \le C \le cl(B)$ and cl(B) = 1 - E. So, C is semiopen fuzzy set but it is not ps-ro semiopen fuzzy set on the fts (Y, τ_1) as there exist no ps-ro open fuzzy set U satisfying $U \le C \le ps-cl(U)$. Again, $A \le F \le ps-cl(A)$. So, F is ps-ro semiopen fuzzy set but not semiopen fuzzy set on the fts (Y, τ_1) as there do not exist any fuzzy open set V satisfying $V \le F \le cl(V)$. Similarly, 1 - C is a semiclosed fuzzy set but is not ps-ro semiclosed fuzzy set and 1 - F is ps-ro semiclosed fuzzy set but not semiclosed fuzzy set on the fts (Y, τ_1) .

Remark 2.2. Example (2.2) shows that ps-ro semiopen(ps-ro semiclosed) fuzzy set and semiopen (semiclosed) fuzzy set are totally independent of each other.

Remark 2.3. In Example (2.2), B is fuzzy open set but not ps-ro semiopen fuzzy set on the fts (Y, τ_1) . In Example (2.1), C is ps-ro semiopen fuzzy set but is not fuzzy open on the fts (X, τ) . This proves that ps-ro semiopen (ps-ro semiclosed) fuzzy set and fuzzy open (fuzzy closed) set are totally independent concept.

Theorem 2.1. A fuzzy set A on a fts (X,τ) is ps-ro semiopen fuzzy set iff $A \leq ps$ -cl(ps-int(A)). Proof. Let A be ps-ro semiopen fuzzy set on X. Then, \exists a ps-ro open fuzzy set U such that $U \leq A \leq ps$ -cl(U). As, ps- $int(U) \leq ps$ -int(A), $U \leq ps$ -int(A). So, ps- $cl(U) \leq ps$ -cl(ps-int(A)). This gives, $A \leq ps$ -cl(ps-int(A)). Conversely, let $A \leq ps$ -cl(ps-int(A)). Let ps-int(A) = V. So, $A \leq ps$ -cl(V). Again, ps- $int(A) \leq A$. So, $V \leq A \leq ps$ -cl(V). Hence, A is ps-ro semiopen fuzzy set on X.

Theorem 2.2. In a fts (X, τ) , ps-ro semiopen and ps-ro semiclosed fuzzy sets are complement of each other.

Proof. Let A be a ps-ro semiopen fuzzy set on X, then \exists a ps-ro open fuzzy set V on X such that $V \leq A \leq ps\text{-}cl(V)$. Hence, $1-V \geq 1-A \geq ps\text{-}int(1-V)$. 1-V being ps-ro closed, 1-A is ps-ro semiclosed fuzzy set on X.

Theorem 2.3. A fuzzy set A on a $fts(X,\tau)$ is ps-ro semiclosed fuzzy set iff ps-int(ps-cl(A)) $\leq A$. Proof. Using Theorem (2.1) and Theorem (2.2) the result follows.

Theorem 2.4. Any union of *ps-ro* semiopen fuzzy sets is a *ps-ro* semiopen fuzzy set on a $fts(X,\tau)$.

Proof. Let $\{A_{\alpha} : \alpha \in \Lambda\}$, where Λ is an index set, be a collection of ps-ro semiopen fuzzy sets on X. Then \exists ps-ro open fuzzy set U_{α} such that $U_{\alpha} \leq A_{\alpha} \leq p$ s- $cl(U_{\alpha})$ for each $\alpha \in \Lambda$. Thus, $\forall U_{\alpha} \leq \forall A_{\alpha} \leq \forall p$ s- $cl(U_{\alpha}) \leq p$ s- $cl(\forall U_{\alpha}) \Rightarrow \forall U_{\alpha} \leq \forall A_{\alpha} \leq p$ s- $cl(\forall U_{\alpha})$. Also, $\forall U_{\alpha}$ is ps-ro open fuzzy set. Hence, $\forall A_{\alpha}$ is ps-ro semiopen fuzzy set. This proves the theorem.

Theorem 2.5. Any intersection of ps-ro semiclosed fuzzy sets is ps-ro semiclosed fuzzy set on a fts (X, τ) .

Proof. Let $\{A_{\alpha} : \alpha \in \Lambda\}$, where Λ is a index set, be a collection of ps-ro semiclosed fuzzy sets on X then \exists ps-ro closed fuzzy set U_{α} such that ps-int $(U_{\alpha}) \leq A_{\alpha} \leq U_{\alpha}$ for each α . Thus, \wedge ps-int $(U_{\alpha}) \leq \wedge A_{\alpha} \leq \wedge U_{\alpha}$. Hence, ps-int $(\wedge U_{\alpha}) \leq \wedge A_{\alpha} \leq \wedge U_{\alpha}$. $\wedge U_{\alpha}$ being ps-ro closed fuzzy set, it follows that $\wedge A_{\alpha}$ is ps-ro semiclosed fuzzy set on X.

Definition 2.2. A fuzzy set A on a fts (X,τ) is said to be

(a) ps-ro semi-neighborhood (nbd) of a fuzzy point x_{α} , if there is a ps-ro semiopen fuzzy set B on X such that $x_{\alpha} \in B \leq A$. (b) ps-ro semi-quasi-nbd or ps-ro sq-nbd of a fuzzy point x_{α} , if there is ps-ro semiopen fuzzy set V on X such that $x_{\alpha}qV \leq A$.

Definition 2.3. A fuzzy point x_{α} on a $fts(X,\tau)$ is called ps-semicluster point of a fuzzy set A on X if every ps-ro sq-nbd of x_{α} is q coincident with A. The set of all ps-semicluster points of A is called as ps-semi-closure of A denoted by ps-scl(A).

Theorem 2.6. For fuzzy set A on a $fts(X,\tau)$, ps-scl(A) is the intersection of all ps-ro semiclosed fuzzy sets each containing A.

Theorem 2.7. A fuzzy set A on a $fts(X,\tau)$ is ps-ro semiclosed fuzzy set iff A = ps-scl(A). Proof. Let A be a ps-ro semiclosed fuzzy set on a $fts(X,\tau)$ then by Theorem (2.6), A = ps-scl(A). Conversely, let A = ps-scl(A), then by Theorem (2.5), A is ps-ro semiclosed fuzzy set.

Definition 2.4. In a $fts(X,\tau)$, ps-semi interior (ps-sint, in short) of a fuzzy subset A is defined as the union of all ps-ro semiopen fuzzy set on X contained in A. i.e. $ps\text{-}sint(A) = \bigvee \{B : B \leq A, B \text{ is } ps\text{-}ro \text{ semiopen fuzzy set on } X\}$.

Theorem 2.8. For any fuzzy sets A and B on a fts (X, τ) , the following hold:

- (a) ps-scl(A) is the smallest ps-ro semiclosed fuzzy set containing A.
- (b) ps-scl(0) = 0, ps-scl(1) = 1.
- (c) $A \leq ps\text{-}scl(A)$.
- (d) ps-scl(ps-scl(A)) = ps-scl(A).
- (e) ps-scl(A) < ps-scl(B), if A < B.
- (f) ps- $scl(A \lor B) = ps$ - $scl(A) \lor ps$ -scl(B).
- (g) ps- $scl(A \wedge B) < ps$ - $scl(A) \wedge ps$ -scl(B).
- (h) $ps\text{-}scl(A) \leq ps\text{-}cl(A)$.

Proof. Straightforward and hence omitted.

Theorem 2.9. For any fuzzy sets A and B on a $fts(X,\tau)$, the following hold:

- (a) ps-sint(A) is the largest ps-ro semiopen fuzzy set contained in A.
- (b) ps-sint(0) = 0, ps-sint(1) = 1.
- (c) ps- $sint(A) \leq A$.
- (d) A is ps-ro semiopen fuzzy set iff A = ps-sint(A).
- (e) ps-sint(ps-sint(A)) = ps-sint(A).

- (f) $ps\text{-}sint(A) \leq ps\text{-}sint(B), if A \leq B$.
- (g) $ps\text{-}sint(A \wedge B) = ps\text{-}sint(A) \wedge ps\text{-}sint(B)$.
- (h) $ps\text{-}sint(A \lor B) \ge ps\text{-}sint(A) \lor ps\text{-}sint(B)$.
- (i) ps- $int(A) \le ps$ -sint(A).
- (j) 1 ps-sint(A) = ps-scl(1 A).
- (k) 1 ps-scl(A) = ps-sint(1 A).

Proof. Straightforward and hence omitted.

3 ps-ro fuzzy semicontinuous, semiopen (closed) functions

Definition 3.1. A function f from a fts (X, τ_1) to another fts (Y, τ_2) is called ps-ro fuzzy semicontinuous function if $f^{-1}(A)$ is ps-ro semiopen fuzzy set on X, for each ps-ro open fuzzy set A on Y.

Remark 3.1. Clearly, every ps-ro fuzzy continuous function is ps-ro fuzzy semicontinuous but the converse is not always true is shown by the following Example:

Example 3.1. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by A(a) = 0.1, A(b) = 0.2 and A(c) = 0.2 and $B(x) = 0.3, \forall x \in X$. Let C, D, E and F be fuzzy sets on Y defined by $C(t) = 0.3, \forall t \in Y$, D(x) = 0.3, D(y) = 0.3 and $D(z) = 0.4, E(t) = 0.4, \forall t \in Y$ and F(x) = 0.1, F(y) = 0.1 and F(z) = 0.2. $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D, E, F\}$ are fuzzy topologies on X and Y respectively. Clearly, A is not pseudo regular open fuzzy set for $0.1 \le \alpha < 0.2$ on X. So, the ps-ro topology on X is $\{0, 1, B\}$. Again, D and F are not pseudo regular open fuzzy set for $0.3 \le \alpha < 0.4$ and $0.1 \le \alpha < 0.2$, respectively on Y. So, the ps-ro topology on Y is $\{0, 1, C, E, \}$. Now, ps-cl(B) = 1 - B, where $(1 - B)(x) = 0.7, \forall x \in X$. Define a function f from the fts (X, τ_1) to fts (Y, τ_2) by f(a) = x, f(b) = y and f(c) = z. E is ps-ro open fuzzy set on Y and $f^{-1}(E)(x) = 0.4, \forall x \in X$. So, $B \le f^{-1}(E) \le p$ s-cl(B). Thus, $f^{-1}(E)$ is ps-ro semiopen fuzzy set on X. Similarly, it can be verified that $f^{-1}(U)$ is ps-ro semiopen fuzzy set on X for every ps-ro fuzzy set U on Y. Hence, f is ps-ro fuzzy semicontinuous but is not ps-ro fuzzy continuous as $f^{-1}(E)$ is not ps-ro open fuzzy set on X.

Remark 3.2. In Example (3.1), $F \in \tau_2$ but $f^{-1}(F) \notin \tau_1$, proving that f is ps-ro fuzzy semicontinuous but is not fuzzy continuous.

Remark 3.3. Let f be a fuzzy continuous function from a $fts(X, \tau_1)$ to $fts(Y, \tau_2)$ and A be a ps-ro open fuzzy set on Y. As, every ps-ro open fuzzy sets are fuzzy open, $f^{-1}(A)$ is fuzzy open on X which by Remark (2.3) is not in general fuzzy ps-ro semiopen fuzzy set on X. Hence, a fuzzy continuous function may not be ps-ro fuzzy semicontinuous. In view of this and Remark (3.2), ps-ro fuzzy semicontinuity and fuzzy continuity do not imply each other.

Example 3.2. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by A(a) = 0.1, A(b) = 0.1 and A(c) = 0.2 and B(x) = 0.3, $\forall x \in X$. Let C, D, E and F be fuzzy sets on Y defined by C(t) = 0.3, $\forall t \in Y$, D(x) = 0.3, D(y) = 0.3 and D(z) = 0.4, E(t) = 0.4, E(t

semiopen fuzzy set on X for every fuzzy open set U on Y but f is not ps-ro fuzzy semicontinuous since $f^{-1}(F)$ is not ps-ro semiopen fuzzy set on X.

Remark 3.4. In Example (3.1), $f^{-1}(F)$ is not semiopen fuzzy set on X, showing that f is not fuzzy semicontinuous but is ps-ro fuzzy semicontinuous. Combining this result with Example (3.2), we see that the concept of ps-ro fuzzy semicontinuous and fuzzy semicontinuous functions are totally independent of each other.

Theorem 3.1. A function f from a $fts(X, \tau_1)$ to another $fts(Y, \tau_2)$ is ps-ro fuzzy semicontinuous function iff $f^{-1}(A)$ is ps-ro semiclosed fuzzy set on X, for each ps-ro closed fuzzy set A on Y. Proof. Let f be ps-ro fuzzy semicontinuous and A be ps-ro closed fuzzy set on Y, then $f^{-1}(1-A) = 1 - f^{-1}(A)$ is ps-ro semiopen fuzzy set on X. Thus, $f^{-1}(A)$ is ps-ro semiclosed fuzzy set on X. Converse part can be proved similarly.

Theorem 3.2. Let f be a function from a fts (X, τ_1) to $fts(Y, \tau_2)$. f is ps-ro fuzzy semicontinuous iff for any fuzzy point x_{α} on X and any ps-ro open fuzzy set V on Y with $f(x_{\alpha}) \in V$, there exist ps-ro semiopen fuzzy set V on Y such that $x_{\alpha} \in U$ and $f(U) \leq V$.

Proof. Let f be ps-ro fuzzy semicontinuous. Let x_{α} be any fuzzy point on X and V be any ps-ro fuzzy open set on Y containing $f(x_{\alpha})$. $f^{-1}(V)$ is ps-ro semiopen fuzzy set which contains x_{α} . Taking $U = f^{-1}(V)$, the result follows. Conversely, let the given condition hold and V be any ps-ro open fuzzy set on Y. If $f^{-1}(V) = 0$, then the result is true. If $f^{-1}(V) \neq 0$, then there exist fuzzy point x_{α} in $f^{-1}(V)$. i.e. $f(x_{\alpha}) \in V$ and V is ps-ro open fuzzy set on Y. So, by the given hypothesis \exists ps-ro semiopen fuzzy set $U_{x_{\alpha}}$ on X which contains x_{α} such that $x_{\alpha} \in U_{x_{\alpha}} \leq f^{-1}(V)$. Since x_{α} is arbitrary, taking union of all such relations, we get $f^{-1}(V) = \bigvee \{x_{\alpha} : x_{\alpha} \in f^{-1}(V)\} \leq \bigvee \{U_{x_{\alpha}} : x_{\alpha} \in f^{-1}(V)\} \leq f^{-1}(V)$. So, $\bigvee \{U_{x_{\alpha}} : x_{\alpha} \in f^{-1}(V)\} = f^{-1}(V)$. This shows $f^{-1}(V)$ is ps-ro semiopen fuzzy set. Hence, f is ps-ro fuzzy semicontinuous.

Theorem 3.3. A function f from a $fts(X, \tau_1)$ to $fts(Y, \tau_2)$ is ps-ro fuzzy semicontinuous iff for every fuzzy set A on X, f(ps- $scl(A)) \le ps$ -cl(f(A)).

Proof. Let f be ps-ro fuzzy semicontinuous and A be any fuzzy set on X. Then, $f^{-1}(ps\text{-}cl(f(A)))$ is ps-ro semiclosed fuzzy set on X.

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Now, A \leq f^{-1}(f(A)) \leq f^{-1}(ps\text{-}cl(f(A)))

\Rightarrow ps\text{-}scl(A) \leq ps\text{-}scl(f^{-1}(ps\text{-}cl(f(A)))) = f^{-1}(ps\text{-}cl(f(A)))

\Rightarrow f(ps\text{-}scl(A)) \leq f(f^{-1}(ps\text{-}cl(f(A)))) \leq ps\text{-}cl(f(A))

\Rightarrow f(ps\text{-}scl(A)) \leq ps\text{-}cl(f(A)).
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Conversely, let B be any ps-ro closed fuzzy set on Y, ps-scl $(f^{-1}(B))$ is ps-ro semiclosed fuzzy set on X. By the given condition we have, $f(ps\text{-scl}(f^{-1}(B))) \leq ps\text{-cl}(f(f^{-1}(B))) \leq ps\text{-cl}(B)$

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\Rightarrow f^{-1}f(ps\text{-}scl(f^{-1}(B))) \le f^{-1}(ps\text{-}cl(B))
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 $\Rightarrow ps\text{-}scl(f^{-1}(B)) \leq f^{-1}f(ps\text{-}scl(f^{-1}(B))) \leq f^{-1}(ps\text{-}cl(B))$

 $\Rightarrow ps\text{-}scl(f^{-1}(B)) \leq f^{-1}(ps\text{-}cl(B)) = f^{-1}(B)$. We have, $ps\text{-}scl(f^{-1}(B)) \geq f^{-1}(B)$. Hence we get, $ps\text{-}scl(f^{-1}(B)) = f^{-1}(B)$. Therefore, $f^{-1}(B)$ is ps-ro semiclosed fuzzy set on X. Hence, f is ps-ro fuzzy semicontinuous.

Theorem 3.4. A function f from a $fts(X, \tau_1)$ to $fts(Y, \tau_2)$ is ps-ro fuzzy semicontinuous iff for every fuzzy set B on Y, ps- $scl(f^{-1}(B)) \le f^{-1}(ps$ -cl(B)).

Proof. Suppose f is ps-ro fuzzy semicontinuous and B be any fuzzy set on Y. Then, $f^{-1}(ps\text{-}cl(B))$ is ps-ro semiclosed fuzzy set on X. Now, $B \leq ps\text{-}cl(B) \Rightarrow f^{-1}(B) \leq f^{-1}(ps\text{-}cl(B))$

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\Rightarrow ps\text{-}scl(f^{-1}(B)) \leq ps\text{-}scl((f^{-1}(ps\text{-}cl(B)))) = f^{-1}(ps\text{-}cl(B))
\Rightarrow ps\text{-}scl(f^{-1}(B))) \leq f^{-1}(ps\text{-}cl(B)). \ \ Conversely, \ let \ B \ be \ any \ ps\text{-}ro \ closed \ fuzzy \ set \ on \ Y. \ \ Given,
ps\text{-}scl(f^{-1}(B)) \leq f^{-1}(ps\text{-}cl(B)) = f^{-1}(B). \ \ So, \ ps\text{-}scl(f^{-1}(B)) \leq f^{-1}(B) \ \ and \ \ f^{-1}(B) \leq ps\text{-}scl(f^{-1}(B)).
Hence, \ f \ is \ ps\text{-}ro \ fuzzy \ semicontinuous.
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Theorem 3.5. For a function f from a fts (X, τ_1) to fts (Y, τ_2) , the following statements are equivalent:

- (a) f is ps-ro fuzzy semicontinuous.
- (b) $ps\text{-}int(ps\text{-}cl(f^{-1}(B))) \leq f^{-1}(ps\text{-}cl(B))$, for each fuzzy set B on Y.
- (c) $f(ps\text{-}int(ps\text{-}cl(A))) \leq ps\text{-}cl(f(A))$, for each fuzzy set A on X.
- $Proof.(a) \Rightarrow (b)$ Suppose f is ps-ro fuzzy semicontinuous and B be any fuzzy set on Y. So, $f^{-1}(ps-cl(B))$ is ps-ro semiclosed fuzzy set on X. Hence, $ps-int(ps-cl(f^{-1}(ps-cl(B)))) \leq f^{-1}(ps-cl(B))$. Thus, $ps-int(ps-cl(f^{-1}(B))) \leq f^{-1}(ps-cl(B))$
- $(b) \Rightarrow (c)$ Let A be any fuzzy set on X. Let f(A) = B, then $A \leq f^{-1}(B)$, Therefore, ps-int(ps- $cl(f^{-1}(B))) \leq f^{-1}(ps-cl(B))$. So, ps-int(ps- $cl(A)) \leq f^{-1}(ps-cl(B))$
- $\Rightarrow f(ps\text{-}int(ps\text{-}cl(A))) \leq f(f^{-1}(ps\text{-}cl(B))) \leq ps\text{-}cl(B) = ps\text{-}cl(f(A)). \ Hence, \ f(ps\text{-}int(ps\text{-}cl(A))) \leq ps\text{-}cl(f(A)).$
- $(c)\Rightarrow (a)$ Suppose U be any ps-ro closed fuzzy set on Y. Let $A=f^{-1}(U)$, then $f(A)\leq U$ and $f(ps-int(ps-cl(A)))\leq ps-cl(f(A))\leq ps-cl(U)=U$
- $\Rightarrow f(ps\text{-}int(ps\text{-}cl(A))) \leq U$
- $\Rightarrow f^{-1}f(ps\text{-}int(ps\text{-}cl(A))) \le f^{-1}(U)$
- $\Rightarrow ps\text{-}int(ps\text{-}cl(A)) \leq f^{-1}f(ps\text{-}int(ps\text{-}cl(A))) \leq f^{-1}(U)$
- $\Rightarrow ps\text{-}int(ps\text{-}cl(f^{-1}(U))) \leq (ps\text{-}cl(A)) \leq f^{-1}(U)$
- So, $f^{-1}(U)$ is ps-ro semiclosed fuzzy set on X. Hence, f is ps-ro fuzzy semicontinuous.

Theorem 3.6. A function f from a fts (X, τ_1) to fts (Y, τ_2) is ps-ro fuzzy semicontinuous iff for every fuzzy set B on Y, $f^{-1}(ps$ - $int(B)) \leq ps$ - $sint(f^{-1}(B))$.

Proof. Let f be ps-ro fuzzy semicontinuous and B be any fuzzy set on Y. Then, $f^{-1}(ps\text{-}int(B))$ is ps-ro semiopen fuzzy set on X. Now, ps-int(B) $\leq B$. So, we get $f^{-1}(ps\text{-}int(B)) = ps\text{-}sint(f^{-1}(ps\text{-}int(B))) \leq ps\text{-}sint(f^{-1}(B))$. Conversely, let B be any ps-ro open fuzzy set on Y. Then, $f^{-1}(ps\text{-}int(B)) = f^{-1}(B) \leq ps\text{-}sint(f^{-1}(B))$ and $f^{-1}(B) \geq ps\text{-}sint(f^{-1}(B))$, therefore, $f^{-1}(B) = ps\text{-}sint(f^{-1}(B))$. Hence, f is ps-ro fuzzy semicontinuous.

Theorem 3.7. Let a function f from a fts (X, τ_1) to fts (Y, τ_1) be one-to-one and onto. f is ps-ro fuzzy semicontinuous iff for every fuzzy set A on X, ps-intf(A)) leq f(ps-sint(A). Proof. Let f be ps-ro fuzzy semicontinuous and A be any fuzzy set on X. Then, $f^{-1}(ps$ -int(f(A)) is ps-ro semiopen fuzzy set on X. Since, f is one-to-one, $f^{-1}(ps$ -int(f(A)) leq ps- $sint(f^{-1}(f(A))) = ps$ -sint(A), i.e. $ff^{-1}(ps$ -int(f(A)) leq f(ps-sint(A)). Since f is onto, ps-int(f(A)) leq f(ps-sint(A). Conversely, let B be any ps-ro open fuzzy set on Y. Since f is onto, B = ps-int(B) = ps- $int(f(f^{-1}(B))) leq f(ps$ - $sint(f^{-1}(B)))$, i.e. $f^{-1}(B) leq f^{-1}(f(ps$ - $sint(f^{-1}(B)))$. Again, f is one-to-one so, $f^{-1}(B) leq ps$ - $sint(f^{-1}(B))$ and $f^{-1}(B) leq ps$ - $sint(f^{-1}(B))$. Therefore, $f^{-1}(B) = ps$ - $sint(f^{-1}(B))$. Hence f is ps-ro fuzzy semicontinuous.

Lemma 3.1. [1] Let $g: X \to X \times Y$ be the graph of a function $f: X \to Y$. If A and B are fuzzy sets on X and Y respectively, then $g^{-1}(A \times B) = A \wedge f^{-1}(B)$.

X. Hence f is ps-ro fuzzy semicontinuous.

Theorem 3.8. Let f be a function from a $fts(X, \tau_1)$ to $fts(Y, \tau_2)$. If the graph $g: X \to X \times Y$ of f is ps-ro fuzzy semicontinuous then f is also ps-ro fuzzy semicontinuous. Proof. Let B be any ps-ro open fuzzy set on Y. Using Lemma (3.1), we get, $f^{-1}(B) = 1 \land f^{-1}(B) = g^{-1}(1 \times B)$. $1 \times B$ is ps-ro open fuzzy set on $X \times Y$ and since g is ps-ro fuzzy semicontinuous function, $g^{-1}(\times B)$ is ps-ro semiopen fuzzy set on X. Thus, $f^{-1}(B)$ is ps-ro semiopen fuzzy set on

Definition 3.2. A function f from a $fts(X, \tau_1)$ to another $fts(Y, \tau_2)$ is called

- (a) ps-ro fuzzy semiopen function if f(A) is ps-ro semiopen fuzzy set on Y for each ps-ro open fuzzy set A on X.
- (b) ps-ro fuzzy semiclosed function if f(A) is ps-ro semiclosed fuzzy set on Y for each ps-ro closed fuzzy set A on X.

Example 3.3. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by A(a) = 0.1, A(b) = 0.2 and A(c) = 0.2 and B(x) = 0.3, $\forall x \in X$. Let C, D, E and F be fuzzy sets on Y defined by C(t) = 0.3, $\forall t \in Y$, D(x) = 0.3, D(y) = 0.3 and D(z) = 0.4, E(t) = 0.4, $\forall t \in Y$ and F(t) = 0.2, $\forall t \in Y$. $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D, E, F\}$ are fuzzy topologies on X and Y respectively. Clearly, A is not pseudo regular open fuzzy set for $0.1 \le \alpha < 0.2$ on X. So, the ps-ro topology on X is $\{0, 1, B\}$. Again, D is not pseudo regular open fuzzy set for $0.3 \le \alpha < 0.4$ on Y. So, the ps-ro topology on Y is $\{0, 1, C, E, F\}$. Define a function f from the fts (X, τ_1) to fts (Y, τ_2) by f(a) = x, f(b) = y and f(c) = z. Here, f is ps-ro fuzzy semiopen but f is neither fuzzy semiopen nor fuzzy open since f(A) is neither semiopen nor open fuzzy set on Y. Here, f is ps-ro fuzzy semiclosed but f is neither fuzzy semiclosed nor fuzzy closed since f(1 - A) is neither semiclosed nor closed fuzzy set on Y.

Example 3.4. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by A(a) = 0.3, A(b) = 0.3 and A(c) = 0.4 and B(x) = 0.3, $\forall x \in X$. Let C and D be two fuzzy sets on Y defined by C(t) = 0.4, $\forall t \in Y$ and D(x) = 0.2, D(y) = 0.3 and D(z) = 0.3. $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D\}$ are fuzzy topologies on X and Y respectively. Clearly, A is not pseudo regular open fuzzy set for $0.3 \le \alpha < 0.4$ on X. So, the ps-ro topology on X is $\{0, 1, B\}$. Again, D is not pseudo regular open fuzzy set for $0.2 \le \alpha < 0.3$ on Y. So, the ps-ro topology on Y is $\{0, 1, C\}$. Define a function f from the fts (X, τ_1) to fts (Y, τ_2) by f(a) = x, f(b) = y and f(c) = z. Here, f is fuzzy semiopen but f is not ps-ro fuzzy semiopen since f(B) is not ps-ro semiopen fuzzy set on Y. Again, f is fuzzy semiclosed but f is not ps-ro fuzzy semiclosed since f(1 - B) is not ps-ro semiclosed fuzzy set on Y.

Remark 3.5. From Remark(3.3) and Remark(3.4), ps-ro fuzzy semiopen(closed) and fuzzy semiopen(closed) functions do not imply each other.

Theorem 3.9. Let f be a function from a $fts(X, \tau_1)$ to $fts(Y, \tau_2)$. f is ps-ro fuzzy semiopen iff f(ps-int $(A)) \leq ps$ -cl(ps-int(f(A))), for any fuzzy set A on X. Proof. Let f be ps-ro fuzzy semiopen and A be a fuzzy set on X. ps-int(A) being ps-ro open fuzzy set on X, f(ps-int(A)) is ps-ro semiopen fuzzy set on Y. By Theorem (2.1), we have, f(ps-int $(A)) \leq ps$ -cl(ps-int(f(ps-int $(A))) \leq ps$ -cl(ps-int(f(A))). Conversely, let A be any ps-ro open fuzzy set on X. f(A) = f(ps-int $(A)) \leq ps$ -cl(ps-int(f(A)) i.e. $f(A) \leq ps$ -cl(ps-int(f(A))). By Theorem (2.1), f(A) is ps-ro semiopen fuzzy set on Y. Hence $f(a) \leq ps$ -ro fuzzy semiopen. **Theorem 3.10.** For a function f from a fts (X, τ_1) to fts (Y, τ_2) , the following statement are equivalent:

- (a) f is ps-ro fuzzy semiopen.
- (b) $f(ps\text{-}int(A)) \leq ps\text{-}sint(f(A))$, for each fuzzy set A on X.
- (c) $ps\text{-}int(f^{-1}(B)) \leq f^{-1}(ps\text{-}sint(B))$, for each fuzzy set B on Y.
- $Proof.(a) \Rightarrow (b)$ Let A be any fuzzy set on X, then f(ps-int(A)) is ps-ro semioprn fuzzy set. Now, $ps\text{-}int(A) \leq A$, so we get $f(ps\text{-}int(A)) = ps\text{-}sint(f(ps\text{-}int(A))) \leq ps\text{-}sint(f(A))$.
- (b) \Rightarrow (c) Let B be any fuzzy set on Y. Then, $f(ps\text{-}int(f^{-1}(B))) \leq ps\text{-}sint(f(f^{-1}(B))) \leq ps\text{-}sint(B)$. Thus, $ps\text{-}int(f^{-1}(B))) \leq f^{-1}(ps\text{-}sint(B))$. Therefore, $ps\text{-}int(f^{-1}(B))) \leq f^{-1}(ps\text{-}sint(B))$.
- (c) \Rightarrow (a) let A be any ps-ro open fuzzy set on X. As, $A \leq f^{-1}f(A)$ we have, $A = ps\text{-}int(A) \leq ps\text{-}int(f^{-1}f(A)) \leq f^{-1}(ps\text{-}sint(f(A)))$. So, $f(A) \leq ps\text{-}sint(f(A))$ and $f(A) \leq ps\text{-}sint(f(A))$. Hence, f(A) = ps-sint(f(A)) i.e. f is ps-ro fuzzy semiopen.

Theorem 3.11. For a function f from a fts (X, τ_1) to fts (Y, τ_2) the following statements are equivalent:

- (a) f is ps-ro fuzzy semiclosed.
- (b) $f(ps-cl(A)) \ge ps-int(ps-cl(f(A)))$, for any fuzzy set A on X.
- (c) ps- $scl(f(A)) \le f(ps$ -cl(A)), for any fuzzy set A on X.
- $Proof.(a) \Rightarrow (b)$ Let A be any fuzzy set on X, f(ps-cl(A)) is ps-ro semiclosed fuzzy set on Y. Now by Theorem (2.3), $ps\text{-}int(ps\text{-}cl(f(ps\text{-}cl(A)))) \leq f(ps\text{-}cl(A))$. Hence, $ps\text{-}int(ps\text{-}cl(f(A))) \leq ps\text{-}int(ps\text{-}cl(f(A))) \leq f(ps\text{-}cl(A))$.
- $(b) \Rightarrow (a)$ Let A be any ps-ro closed fuzzy set on X. ps-int(ps-cl(f(A))) $\leq f(ps\text{-cl}(A)) = f(A)$.ie. f(A) is ps-ro semiclosed fuzzy set on Y. Hence, f is ps-ro fuzzy semiclosed.
- $(a) \Rightarrow (c)$ Let A be any fuzzy set on X. f(ps-cl(A)) is ps-ro semiclosed fuzzy set on Y. $f(A) \leq f(ps\text{-}cl(A))$. So, $ps\text{-}scl(f(A)) \leq ps\text{-}scl(f(ps\text{-}cl(A))) = f(ps\text{-}cl(A))$. Hence, $ps\text{-}scl(f(A)) \leq f(ps\text{-}cl(A))$. $(c) \Rightarrow (a)$ Let A be any ps-ro closed fuzzy set on X. Now, $ps\text{-}scl(f(A)) \leq f(ps\text{-}cl(A)) = f(A)$. We have $f(A) \leq ps\text{-}scl(f(A))$. Hence, f(A) = ps-scl(f(A)). i.e. f(A) is ps-ro semiclosed fuzzy set on
- Y. Therefore, f is ps-ro fuzzy semiclosed.

Theorem 3.12. Let a function f from a fts (X, τ_1) to fts (Y, τ_1) be one-to-one and onto. f is ps-ro fuzzy semiclosed iff for every fuzzy set B on Y, $f^{-1}(ps$ - $scl(B)) \leq ps$ - $cl(f^{-1}(B))$. Proof. Let f be ps-ro fuzzy semiclosed and B be any fuzzy set on Y. As f is onto, ps-scl(B) = ps- $scl(f(f^{-1}(B))) \leq f(ps$ - $cl(f^{-1}(B))$, i.e $f^{-1}(ps$ - $scl(B)) \leq f^{-1}(f(ps$ - $cl(f^{-1}(B)))$. Since, f is one-to-one we get $f^{-1}(ps$ - $scl(B)) \leq ps$ - $cl(f^{-1}(B))$. Conversely, let A be any ps-ro closed fuzzy set on X. Since, f is one-to-one, $f^{-1}(ps$ - $scl(f(A)) \leq ps$ - $cl(f^{-1}(f(A))) = ps$ -cl(A) = A, i.e. $f(f^{-1}(ps$ - $scl(f(A))) \leq f(A)$. Since, f is onto ps- $scl(f(A)) \leq f(A)$ and ps- $scl(f(A)) \geq f(A)$. Therefore, ps-scl(f(A)) = f(A). Hence, f is ps-ro fuzzy semiclosed.

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